Interpolation, Patterns and Terrain Modeling
Interpolation Method: Purpose and Principles

“We are not able to measure the values of studied phenomena everywhere in a space, but only in sample points. Therefore we need to create continuous surfaces to obtain values of the studied phenomenon anywhere.”
Interpolation creates continuous surfaces for data, filling “voids” where data were not collected. It helps to estimate unknown values from available known values within their spatial extent.

After continuous surface is created it has “patterns” that scientists try to understand. Visual information is processed faster than numeric one.
Interpolation Methods: **Products**

Visualization is a graphical or *virtual* representation of the interpolated data.

1. Contour Lines
2. 2D and 3D Surfaces
3. Shaded, perspective surfaces.
4. Virtual Reality
Pattern definitions from the WorldNet ([https://wordnet.princeton.edu/](https://wordnet.princeton.edu/)):

1. form, shape, pattern -- (a perceptual structure; "the composition presents problems for students of musical form"; "a visual pattern must include not only objects but the spaces between them")

2. design, pattern, figure -- (a decorative or artistic work; "the coach had a design on the doors")

4. convention, normal, pattern, rule, formula -- (something regarded as a normative example; "the convention of not naming the main character"; "violence is the rule not the exception"; "his formula for impressing visitors")

5. pattern -- (a model considered worthy of imitation; "the American constitution has provided a pattern for many republics")

6. blueprint, design, pattern -- (something intended as a guide for making something else; "a blueprint for a house"; "a pattern for a skirt")

7. radiation pattern, radiation diagram, pattern -- (graphical representation (in polar or cartesian coordinates) of the spatial distribution of radiation from an antenna as a function of angle)

Discovered patterns in data help us to ask question: why?

Data visualization (including interpolated surfaces) helps us to see patterns.
Paleoclimatic Reconstruction patterns:

Contour line pattern created from collected sampling sites shows a relative position of Equilibrium Line Altitudes (ELA) in Disko Island, West Greenland.

Results from pattern analysis:

“In the easternmost part of the island the vertical shift in ELA is 400-500m. In the northwestern part they are as low as 100 m.

This pattern cannot be result of regional changes in temperature …”

Crime mapping technique:

“HOT SPOTS” pattern (variety of density analysis)

Recovery of guns in Staten Island and Brooklyn, 1999 - 2003

Number of recoveries per sq. mile

Density:
- 0 - 200
- 200 - 400
- 400 - 600
- 600 - 800
- 800 - 1000
- 1000 - 1200
- 1200 - 1400
- >1400

Dark color shows high recovery rate.

Boundaries help outline areas of existing and potential recoveries; can identify areas that would require more attention from police.

Data Source: ATFE, West Virginia
Watershed boundary delineation using contour lines:

Pattern of contour lines helps to identify watershed boundary

Dense contour line pattern means steep terrain.

“Spacious” contour line pattern means flat terrain
Influence of data structure on patterns in modeling surface:

Difference in patterns is due to the data structure, NOT data itself

Main Interpolation Methods

Vector Based:

1. Triangulated Irregular Network (TIN)
2. Voronoi (Thiessen) Polygons aka Area Weighted Method.

Raster Based:

1. Inverse Distance Weighted
2. Spline
3. Kriging
Vector Interpolation Methods:

Voronoy aka Dirichlet aka Thiessen

Dirichlet Johann Peter Gustav Lejeune (1805 – 1859), french/german mathematician.

<Weight of the value has its own spatial limit>

Voronoy Georgii Fedoseevich (1868-1908),
russian mathematician.

<Build mathematical algorithm while study crystallography>

Thiessen A.D. (1872 - 1956), american meteorologist.

<First practical application in meteorology>

Vector Interpolation Methods: Voronoy

Choropleth Map as a result of the Voronoy interpolation.

Red dots are original data points.

Colors represent various values.

Conclusions:

1. The appearance of the final map is determined by the distribution of the observations.

2. The method maintains the choropleth map fiction of homogeneity within borders and all change at borders.

3. Because all predictions equal the values at the data points, Voronoy polygons are exact predictors.
Thiessen Method for Average Rainfall

Step 1
Thiessen Method for Average Rain

Step 2
Thiessen Method for Average Rain

(A) P = 1.81"
    P = 2.15"
    P = 2.26"
    P = 2.18"
    P = 1.62"
    P = 1.80"

(B) P = 1.81"
    P = 2.15"
    P = 2.26"
    P = 2.18"
    P = 1.62"
    P = 1.80"

(C) P = 1.81"
    P = 2.15"
    P = 2.26"
    P = 2.18"
    P = 1.62"
    P = 1.80"

Step 3
Thiessen Method for Average Rain

Step 4
The average rainfall for the entire area is then assumed to be a weighted average of the observed rainfalls, calculated by equation:

\[
P = \frac{\sum_{i=1}^{n} A_i P_i}{\sum_{i=1}^{n} A_i}
\]

where \( P \) represents the average depth of rainfall in the watershed with a total area of \( \Sigma \) and \( A_i \) is the area of the \( i \)th polygon with precipitation of \( P_i \) in that polygon.

\[
P = \frac{(65 \times 1.81) + (150 \times 2.15) + (269 \times 2.26) + (216 \times 2.18) + (56 \times 1.62) + (136 \times 1.8)}{(65 + 150 + 269 + 216 + 56 + 136)} = 2.08 \text{ inches}
\]
Thiessen Method in ArcGIS

Vector Interpolation Method: also known as Voronoy or Thiessen

Summary

Creates Thiessen polygons from point input features.

Each Thiessen polygon contains only a single point input feature. Any location within a Thiessen polygon is closer to its associated point than to any other point input feature.

Illustration
Thiessen Method in ArcGIS

Create Thiessen Polygons

Creates Thiessen polygons from point input features.

Each Thiessen polygon contains only a single point input feature. Any location within a Thiessen polygon is closer to its associated point than to any other point input feature.
Vector Interpolation Methods: **Delaunay (Delone) aka TINs**

Delauney Boris Nikolaevich (1890 - 1980), russian mathematician

Delaunay triangulation for any set of points in two dimensions is always unique as long as no four points in the point set are co-circular. Because it minimizes small angles and circumscribed circles, the Delaunay triangulation is geometrically nice and, in general, pleasing to the eye.
Ways of Connecting Points
Law of Triangulation

To describe surface we need minimum three points, i.e. A, B and C. AB is an edge of the Delaunay triangulation if there is a circle passing through A and B so that all other points in the point set besides C, where C is not equal to A or B, lie outside the circle.

Equivalently, all triangles in the Delaunay triangulation for a set of points will have empty circumscribed circles. **No points should lie in the interior of any triangle's circumcircle.**
Interpolation Methods: TIN

Conclusions:

1. Fast and efficient way to create a non-redundant (unlike raster data) continuous distribution surface of the specific parameter.

2. TIN is a perfect tool to create a regular raster matrix or lattice. The final matrix or lattice can provide a basis for the further interpolation.

black lines represent TIN
yellow – raster dataset
Raster Interpolation Methods:

Inverse Distance Weighted (aka Weighting or IDW).

• The method is local i.e. it works on a specific point and surrounding neighborhood of points or point clusters, located either randomly or regularly.

• The method is based on assumption that the influence of point value reduces with the distance to its neighbors.
**Raster Interpolation Methods:**

**IDW Technique.**

Normally, it is useful to constrain computation to points in a neighborhood of the location for which we wish to obtain a value.

\[ x = \frac{\sum_{i=1}^{n} (Z_i/D_i)}{\sum_{i=1}^{n} \left(\frac{1}{D_i}\right)} \]

Where,
- \( x \) = interpolated value,
- \( Z_i \) = data value, of which there are \( n \) in the neighborhood of \( Z \),
- \( D_i \) = distance between \( x \) and each data point.

Intuitively, this represents the average of the values of the surrounding points, weighted by the inverse of the distance to those points.
Interpolation Methods: Raster

Spline.

This method uses input points as a rigid framework to fit a minimum curvature surface. Input points preserve their location and values. Original idea came from the drafting trade, where splines (drafting tools, consisting of weights and a string) were used to create a smooth surface around selected points.
Raster Interpolation Methods:

Kriging (After D.G.Krige, geologist who worked in mining industry in South Africa, 60s-70s)

This method estimates the unknown values with minimum variances.

First step in kriging is calculation of variogram, i.e. semi-variances for the distance class between sampling points.

Second step deals with approximating of the semi-variograms with functions fitted to the experimental data.

Third step is creation of the surface.
**Raster Interpolation: Kernel Smoothing ("cousin" of Density function)**

**STEPS:**
- The grid overlay the study area (i.e. select pixel size).
- The circular window with a fixed radius (bandwidth) is moving across the grid passing through each grid point (cell).
- The density of events is computed within the window and weighted according to their distance from the center of the window. (IDW principle).

**PROBLEMS:**
- Choice of the bandwidth.
- Choice of the threshold value for the “hot spot”.
- Boundary condition.
- Pixel size selection.
- Kernel Density value is difficult to interpret

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Raster Interpolation: Kernel Smoothing (variety of Density function)

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raw data

Interpolated data
Terrain Modeling and Analysis

All Terrain Analysis Functions in ArcGIS are in ArcToolbox → Spatial Analyst Tools.

To invoke its license: Customize → Extensions → check box next to Spatial Analyst
Inverse Distance Weighted Interpolation

Variable Radius (min 12 points)

Interpolates a raster surface from points using an inverse distance weighted (IDW) technique.

Elevation range: 1300 – 3160
Inverse Distance Weighted Interpolation

Fixed Radius (150 m)

Input point features
- ELEVATION
- Output raster
- Output cell size (optional): 20
- Power (optional): 2
- Search radius (optional): Fixed
- Distance: 150
- Minimum number of points: 

Input point features
The input point features containing the z-values to be interpolated into a surface raster.
Inverse Distance Weighted Interpolation: differences in sampling method

Variable – 12 points

Fixed - 150 m

Fixed -500 m

Fixed - 800 m
Spline

Regularized

Tension

Elevation range: 1300 - 3160
Spline: influence of weight on values range

Weight = 0.1

Weight = 30
INTERPOLATION METHODS COMPARISON

IDW, variable, 12 points

Spline, tension, weight = 30

Elevation range: 1300 - 3160
Spatial filtering or application of focal functions:

In ArcGIS it can be found in: ArcToolbox → Spatial Analyst Tools → Neighborhood

These functions are applied for grids to:

- smooth elevation
- extract edges
- improve visualization

*Burrough and McDonnel, 1998. Principles of Geographic Information Systems*
**Spatial Filtering** (smoothing, edge detection, etc.)

Mean:

Focal Statistics tool with Mean option

3 x 3 neighborhood

7 x 7 neighborhood
Contour Line Generation:

ArcToolbox → Spatial Analyst
Tools → Surface → Contour
Because slope uses vertical and horizontal measures pay attention to units. Horizontal units depend on spatial reference units (e.g. decimal degree or meters), vertical units depend on elevation units (i.e. meters, feet). **If vertical and horizontal units are not the same (i.e. inconsistent), slope value will be wrong.**
Aspect Generation:

Aspect

Derives aspect from a raster surface. The aspect identifies the downslope direction of the maximum rate of change in value from each cell to its neighbors.

Aspect can be thought of as the slope direction. The values of the output raster will be the compass direction of the aspect.
Hillshade Generation:

Hillshade

Creates a shaded relief from a surface raster by considering the illumination source angle and shadows.
Hillshade and DEM visualization “secret”:

1. “Spline of elevations” → Properties: set transparency value

2. Place spline of elevation layer above Hillshade
Viewshed Analysis: helps mapping visibility areas

- Viewer at low elevation
- Viewer at high elevation

Visibility areas or Viewshed (purple color)

**Viewshed**
Determines the raster surface locations visible to a set of observer features.
Terrain Analysis: Profile (using extension 3D Analyst)

- Create Line
- Create Profile

Profile Graph Title

Profile Graph Subtitle
Terrain Analysis: Profile (using extension Easy Profile 9.03)
Terrain Analysis: 3D Visualization with ArcScene

Settings: Layer Properties → Base Heights
Base height is either constant value or surface (i.e. interpolated data)
Visualizing Continuous Data with “Stretched” and “Classified” options